

Quantum effect of one-dimensional photonic crystal

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In this paper, we have studied the quantum transmission characteristics of one-dimensional photonic crystal with and without defect layer by the quantum theory approach, and compared the calculation results of classical with quantum theory. We have found some new quantum effects in the one-dimensional photonic crystal. When the incident angle $\theta = 0$, there is no quantum effect. When the incident angle $\theta \neq 0$, we find there are obvious quantum effect with the incident angle increase. At the incident angle $\theta \neq 0$, there are also quantum effect with the change of thickness and refractive indexes of medium B or A . We think the new quantum effect of photonic crystal shall help us to design optical devices.

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1. Introduction

In 1987, E. Yablonovitch and S. John had pointed out that the behavior of photons. It can be changed when propagating in the material with periodical dielectric constant, and termed such material Photonic Crystal [1, 2]. Photonic crystal important characteristics are: Photon Band Gap, defect states, Light Localization and so on. These characteristics make it able to control photons, so it may be used to manufacture some high performance devices which have completely new principles or can not be manufactured before, such as high-efficiency semiconductor lasers, light emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical wave guides and sharp bends [3, 4], WDM-devices [5, 6], splitters and combiners [7]. optical limiters and amplifiers [8-10]. The research on photonic crystals will promote its application and development on integrated photoelectron devices and optical communication. To investigate the structure and characteristics of band gap, there are many methods to analyze Photonic crystals including the plane-wave expansion method [11], Greens function method, finite-difference time-domain method [12-14] and transfer matrix method [15-20]. All of methods are come from classical Maxwell equations. In Refs. [21, 22], we have firstly studied the the quantum transmission characteristics of one-dimensional photonic crystal by the quantum theory approach, in which we have only considered the incident angle is zero, i.e., vertical incidence, we have found the classical and quantum transmission characteristics are the completely same, i.e., there is not quantum effect in one-dimensional photonic crystal. In this paper, we have studied the quantum transmission characteristics of one-dimensional photonic crystal when the incident angle is an arbitrary angle, i.e., non-vertical incidence. we find there are obvious quantum effect with the incident angle increase. At the incident angle $\theta \neq 0$, there are also obvious quantum effect with the change of thickness and refractive indexes of medium B or A . Otherwise, we have considered the effect of defect layer on the quantum transmission characteristics. When the incident angle $\theta = 0$, there is also not quantum effect. When the incident angle $\theta \neq 0$, with the incident angle increase, there are obvious quantum effect for the one-dimensional photonic crystal with defect layer.

2. The quantum transmissivity

In Refs. [23, 24], with the quantum theory approach, we have studied one-dimensional photonic crystal quantum transmissison characteristic when the incident photon is vertical incidence, i.e., the incident angle $\theta = 0$. In the paper, we shall study one-dimensional photonic crystal quantum transmissison characteristic

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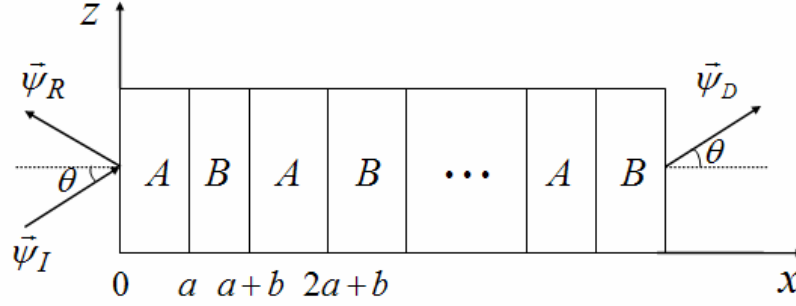


FIG. 1: The quantum structure of one-dimensional photonic crystal.

when the incident photon is non-vertical incidence, i.e., the incident angle $\theta \neq 0$.

The quantum wave equations of photon in medium is [23, 24]

$$\nabla \times \vec{\psi} = \frac{E - V}{c\hbar} \vec{\psi} = \frac{\omega}{c} n \vec{\psi}, \quad (1)$$

where ω is angle frequency of photon, c is the velocity of photon, $\vec{\psi}$ is the wave function of photon and n is refractive index of medium.

The incident light, reflected light and transmission light are in the xz plane, the incident angle is θ , which are shown in Fig. 1. The wave vectors K_I and K_R of incident and reflection photon are

$$\vec{K}_I = K_x \vec{i} + K_z \vec{k}, \quad \vec{K}_R = -K_x \vec{i} + K_z \vec{k} \quad (2)$$

where $K_x = K \cos \theta$, $K_z = K \sin \theta$ are the wave vectors in x and z direction, and $K = \frac{\omega}{c}$, the ω is the incident photon angle frequency. The wave functions $\vec{\psi}_I$ and $\vec{\psi}_R$ of incident and reflection photon can be written as

$$\vec{\psi}_I = F_x e^{i(K_x x + K_z z)} \vec{i} + F_y e^{i(K_x x + K_z z)} \vec{j} + F_z e^{i(K_x x + K_z z)} \vec{k}, \quad (3)$$

$$\vec{\psi}_R = F'_x e^{i(-K_x x + K_z z)} \vec{i} + F'_y e^{i(-K_x x + K_z z)} \vec{j} + F'_z e^{i(-K_x x + K_z z)} \vec{k}, \quad (4)$$

where F_x (F'_x), F_y (F'_y) and F_z (F'_z) are the amplitudes of incident (reflected) photon wave functions in the x , y and z directions.

Substituting $\vec{\psi}_I$ into Eq. (1), we have

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x e^{i(K_x x + K_z z)} & F_y e^{i(K_x x + K_z z)} & F_z e^{i(K_x x + K_z z)} \end{vmatrix} = \frac{\omega}{c} (F_x e^{i(K_x x + K_z z)} \vec{i} + F_y e^{i(K_x x + K_z z)} \vec{j} + F_z e^{i(K_x x + K_z z)} \vec{k}), \quad (5)$$

or

$$\begin{cases} \frac{\partial}{\partial y} F_z e^{i(K_x x + K_z z)} - \frac{\partial}{\partial z} F_y e^{i(K_x x + K_z z)} = \frac{\omega}{c} F_x e^{i(K_x x + K_z z)}, \\ -\frac{\partial}{\partial x} F_z e^{i(K_x x + K_z z)} + \frac{\partial}{\partial z} F_x e^{i(K_x x + K_z z)} = \frac{\omega}{c} F_y e^{i(K_x x + K_z z)}, \\ \frac{\partial}{\partial x} F_y e^{i(K_x x + K_z z)} - \frac{\partial}{\partial y} F_x e^{i(K_x x + K_z z)} = \frac{\omega}{c} F_z e^{i(K_x x + K_z z)}, \end{cases} \quad (6)$$

i.e.,

$$\begin{cases} -iK_z F_y = \frac{\omega}{c} F_x = K F_x, \\ -iK_x F_z + iK_z F_x = \frac{\omega}{c} F_y = K F_y, \\ iK_x F_y = \frac{\omega}{c} F_z = K F_z, \end{cases} \quad (7)$$

From Eq. (7), we get

$$F_x = -\frac{iK_z}{K} F_y, \quad (8)$$

$$-iK_x F_z = (K - \frac{K_z^2}{K}) F_y, \quad (9)$$

and

$$K_x^2 + K_z^2 = K^2. \quad (10)$$

The incident current is [23, 24]

$$\begin{aligned} \vec{J}_I &= ic\vec{\psi}_I \times \vec{\psi}_I^* \\ &= 2c[\frac{K^2 - K_z^2}{K K_x} \vec{i} + \frac{K_z}{K} \vec{k}] \cdot |F_y|^2. \end{aligned} \quad (11)$$

We find the incident current \vec{J} is related to the amplitude F_y of y component. So, we should only consider the \vec{j} component wave function of photon in the following calculation.

Firstly, we study the transfer matrices in the first period. The wave function of photon in medium A is

$$\begin{aligned} \vec{\psi}_A^1 &= [A_x^1 e^{i(K_{Ax}x + K_{Az}z)} + A_x'^1 e^{i(-K_{Ax}x + K_{Az}z)}] \vec{i} \\ &\quad + [A_y^1 e^{i(K_{Ax}x + K_{Az}z)} + A_y'^1 e^{i(-K_{Ax}x + K_{Az}z)}] \vec{j} \\ &\quad + [A_z^1 e^{i(K_{Ax}x + K_{Az}z)} + A_z'^1 e^{i(-K_{Ax}x + K_{Az}z)}] \vec{k}, \end{aligned} \quad (12)$$

where A_x^1 ($A_x'^1$), A_y^1 ($A_y'^1$) and A_z^1 ($A_z'^1$) are the amplitudes of incident (reflected) photon wave functions in the x , y and z directions.

Substituting $\vec{\psi}_A^1$ into Eq. (1), we have

$$-\frac{\partial}{\partial z}(A_y^1 e^{i(K_{Ax}x + K_{Az}z)} + A_y'^1 e^{i(-K_{Ax}x + K_{Az}z)}) = \frac{\omega}{c} n_A (A_x^1 e^{i(K_{Ax}x + K_{Az}z)} + A_x'^1 e^{i(-K_{Ax}x + K_{Az}z)}), \quad (13)$$

$$\begin{aligned} -\frac{\partial}{\partial x}(A_z^1 e^{i(K_{Ax}x + K_{Az}z)} + A_z'^1 e^{i(-K_{Ax}x + K_{Az}z)}) + \frac{\partial}{\partial z}(A_x^1 e^{i(K_{Ax}x + K_{Az}z)} + A_x'^1 e^{i(-K_{Ax}x + K_{Az}z)}) \\ = \frac{\omega}{c} n_A [A_y^1 e^{i(K_{Ax}x + K_{Az}z)} + A_y'^1 e^{i(-K_{Ax}x + K_{Az}z)}], \end{aligned} \quad (14)$$

$$\frac{\partial}{\partial x}(A_y^1 e^{i(K_{Ax}x + K_{Az}z)} + A_y'^1 e^{i(-K_{Ax}x + K_{Az}z)}) = \frac{\omega}{c} n_A [A_z^1 e^{i(K_{Ax}x + K_{Az}z)} + A_z'^1 e^{i(-K_{Ax}x + K_{Az}z)}]. \quad (15)$$

From Eq. (13) to (15), we obtain

$$\frac{A_y^1}{A_y'^1} = \frac{A_x^1}{A_x'^1} = -\frac{A_z^1}{A_z'^1}. \quad (16)$$

In the incidence zone, the total wave function is

$$\begin{aligned}
\vec{\psi}_{tot}(x, y, z) &= \vec{\psi}_I(x, y, z) + \vec{\psi}_R(x, y, z) \\
&= (F_x e^{i(K_x x + K_z z)} + F'_x e^{i(-K_x x + K_z z)}) \vec{i} \\
&\quad + (F_y e^{i(K_x x + K_z z)} + F'_y e^{i(-K_x x + K_z z)}) \vec{j} \\
&\quad + (F_z e^{i(K_x x + K_z z)} + F'_z e^{i(-K_x x + K_z z)}) \vec{k},
\end{aligned} \tag{17}$$

In the following, we should use the condition of wave function and its derivative continuation at interface of two mediums.

(1) At $x = 0$, by the continuation of wave functions $\vec{\psi}_{tot}(x, y, z)$ and $\vec{\psi}_A^1(x, y, z)$, we have

$$\begin{aligned}
&(F_x + F'_x) e^{iK_z z} \vec{i} + (F_y + F'_y) e^{iK_z z} \vec{j} + (F_z + F'_z) e^{iK_z z} \vec{k} \\
&= (A_x^1 + A_x'^1) e^{iK_z z} \vec{i} + (A_y^1 + A_y'^1) e^{iK_z z} \vec{j} + (A_z^1 + A_z'^1) e^{iK_z z} \vec{k},
\end{aligned} \tag{18}$$

(2) At $x = 0$, by the derivative continuation of wave functions $\vec{\psi}_{tot}(x, y, z)$ and $\vec{\psi}_A^1(x, y, z)$, we have

$$\begin{aligned}
&iK_x(F_x - F'_x) e^{iK_z z} \vec{i} + iK_x(F_y - F'_y) e^{iK_z z} \vec{j} \\
&\quad + iK_x(F_z - F'_z) e^{iK_z z} \vec{k} \\
&= iK_{Ax}(A_x^1 - A_x'^1) e^{iK_{Az} z} \vec{i} + iK_{Ax}(A_y^1 - A_y'^1) e^{iK_{Az} z} \vec{j} \\
&\quad + iK_{Ax}(A_z^1 - A_z'^1) e^{iK_{Az} z} \vec{k},
\end{aligned} \tag{19}$$

with Eqs. (18) and (19), we get the \vec{j} component relations

$$(F_y + F'_y) e^{iK_z z} = (A_y^1 + A_y'^1) e^{iK_{Az} z}, \tag{20}$$

and

$$K_x(F_y - F'_y) e^{iK_z z} = K_{Ax}(A_y^1 - A_y'^1) e^{iK_{Az} z}, \tag{21}$$

by Eqs. (20) and (21), we obtain

$$K_z = K_{Az}, \tag{22}$$

and

$$\begin{cases} A_y^1 + A_y'^1 = F_y + F'_y \\ A_y^1 - A_y'^1 = \frac{K_x}{K_{Ax}}(F_y - F'_y) \end{cases}, \tag{23}$$

by Eq. (23), we can obtain the matrix form of A_y^1 , $A_y'^1$ and F_y , F'_y

$$\begin{aligned}
\begin{pmatrix} A_y^1 \\ A_y'^1 \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} 1 + \frac{K_x}{K_{Ax}} & 1 - \frac{K_x}{K_{Ax}} \\ 1 - \frac{K_x}{K_{Ax}} & 1 + \frac{K_x}{K_{Ax}} \end{pmatrix} \begin{pmatrix} F_y \\ F'_y \end{pmatrix} \\
&= M_A^1 \begin{pmatrix} F_y \\ F'_y \end{pmatrix},
\end{aligned} \tag{24}$$

where

$$M_A^1 = \frac{1}{2} \begin{pmatrix} 1 + \frac{K_x}{K_{Ax}} & 1 - \frac{K_x}{K_{Ax}} \\ 1 - \frac{K_x}{K_{Ax}} & 1 + \frac{K_x}{K_{Ax}} \end{pmatrix}, \tag{25}$$

is the transfer matrix of medium A in the first period, and the K_{Ax} is

$$K_{Ax} = \sqrt{K_A^2 - K_{Az}^2} = \sqrt{K_A^2 - K_z^2} = \sqrt{K_A^2 - K^2 \sin^2 \theta}, \quad (26)$$

where $K_A = \frac{\omega}{c} n_a$, n_a is the refractive indexes of medium A , and the transfer matrix M_A^1 can be written as

$$M_A^1 = \frac{1}{2} \begin{pmatrix} 1 + \frac{K \cos \theta}{\sqrt{K_A^2 - K^2 \sin^2 \theta}} & 1 - \frac{K \cos \theta}{\sqrt{K_A^2 - K^2 \sin^2 \theta}} \\ 1 - \frac{K \cos \theta}{\sqrt{K_A^2 - K^2 \sin^2 \theta}} & 1 + \frac{K \cos \theta}{\sqrt{K_A^2 - K^2 \sin^2 \theta}} \end{pmatrix}. \quad (27)$$

The wave function of photon in medium B is

$$\begin{aligned} \vec{\psi}_B^1(x, y, z) &= (B_x^1 e^{i(K_{Bx}x + K_{Bz}z)} + B_x'^1 e^{i(-K_{Bx}x + K_{Bz}z)}) \vec{i} \\ &+ (B_y^1 e^{i(K_{Bx}x + K_{Bz}z)} + B_y'^1 e^{i(-K_{Bx}x + K_{Bz}z)}) \vec{j} \\ &+ (B_z^1 e^{i(K_{Bx}x + K_{Bz}z)} + B_z'^1 e^{i(-K_{Bx}x + K_{Bz}z)}) \vec{k}, \end{aligned} \quad (28)$$

where B_x^1 ($B_x'^1$), B_y^1 ($B_y'^1$) and B_z^1 ($B_z'^1$) are the amplitudes of incident (reflected) photon wave functions in the x , y and z directions.

(3) At $x = a$, by the continuation of \vec{j} component wave functions $\vec{\psi}_A^1(x, y, z)$, $\vec{\psi}_B^1(x, y, z)$ and their derivative, we have

$$(A_y^1 + A_y'^1) e^{iK_{Az}z} + A_y^1 e^{iK_{Ax}a} + A_y'^1 e^{-iK_{Ax}a} = (B_y^1 + B_y'^1) e^{iK_{Bz}z} + B_y^1 e^{iK_{Bx}a} + B_y'^1 e^{-iK_{Bx}a}, \quad (29)$$

and

$$iK_{Ax}(A_y^1 e^{iK_{Ax}a + iK_{Az}z} - A_y'^1 e^{-iK_{Ax}a + iK_{Az}z}) = iK_{Bx}(B_y^1 e^{iK_{Bx}a + iK_{Bz}z} - B_y'^1 e^{-iK_{Bx}a + iK_{Bz}z}), \quad (30)$$

with Eq. (29), we have

$$(A_y^1 + A_y'^1) e^{iK_{Az}z} - (B_y^1 + B_y'^1) e^{iK_{Bz}z} = 0, \quad (31)$$

$$A_y^1 e^{iK_{Az}a} + A_y'^1 e^{-iK_{Az}a} - (B_y^1 e^{iK_{Bx}a} + B_y'^1 e^{-iK_{Bx}a}) = 0, \quad (32)$$

with Eq. (30), we get

$$K_{Ax}(A_y^1 - A_y'^1) e^{iK_{Az}z} - K_{Bx}(B_y^1 - B_y'^1) e^{iK_{Bz}z} = 0, \quad (33)$$

$$K_{Ax}(A_y^1 e^{iK_{Ax}a} - A_y'^1 e^{-iK_{Ax}a}) - K_{Bx}(B_y^1 e^{iK_{Bx}a} + B_y'^1 e^{-iK_{Bx}a}) = 0, \quad (34)$$

with Eqs. (31) and (33), we obtain

$$K_{Az} = K_{Bz}. \quad (35)$$

The Eqs. (32) and (34) can be written as

$$\begin{cases} A_y^1 e^{iK_{Ax}a} + A_y'^1 e^{-iK_{Ax}a} = B_y^1 e^{iK_{Bx}a} + B_y'^1 e^{-iK_{Bx}a} \\ K_{Ax}(A_y^1 e^{iK_{Ax}a} - A_y'^1 e^{-iK_{Ax}a}) = K_{Bx}(B_y^1 e^{iK_{Bx}a} - B_y'^1 e^{-iK_{Bx}a}) \end{cases}. \quad (36)$$

By Eq. (36), we can obtain the matrix form of B_y^1 , $B_y'^1$ and A_y^1 , $A_y'^1$

$$\begin{aligned} \begin{pmatrix} B_y^1 \\ B_y'^1 \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} (1 + K_{Ax}/K_{Bx}) e^{i(K_{Ax} - K_{Bx})a} & (1 - K_{Ax}/K_{Bx}) e^{-i(K_{Bx} + K_{Ax})a} \\ (1 - K_{Ax}/K_{Bx}) e^{i(K_{Ax} + K_{Bx})a} & (1 + K_{Ax}/K_{Bx}) e^{i(K_{Bx} - K_{Ax})a} \end{pmatrix} \begin{pmatrix} A_y^1 \\ A_y'^1 \end{pmatrix} \\ &= M_B^1 \begin{pmatrix} A_y^1 \\ A_y'^1 \end{pmatrix}, \end{aligned} \quad (37)$$

where

$$M_B^1 = \frac{1}{2} \begin{pmatrix} (1 + K_{Ax}/K_{Bx})e^{i(K_{Ax}-K_{Bx})a} & (1 - K_{Ax}/K_{Bx})e^{-i(K_{Bx}+K_{Ax})a} \\ (1 - K_{Ax}/K_{Bx})e^{i(K_{Ax}+K_{Bx})a} & (1 + K_{Ax}/K_{Bx})e^{i(K_{Bx}-K_{Ax})a} \end{pmatrix}, \quad (38)$$

is the transfer matrix of medium B in the first period, and the K_{Bx} is

$$K_{Bx} = \sqrt{K_B^2 - K^2 \sin^2 \theta}, \quad (39)$$

where $K_B = \frac{\omega}{c} n_b$, n_b is the refractive indexes of medium B .

Secondly, we use the similar approach can obtain the transfer matrices M_A^2 and M_B^2 of media A and B in the second period, they are

$$M_A^2 = \frac{1}{2} \begin{pmatrix} (1 + K_B/K_A)e^{i(K_B-K_A)(a+b)} & (1 - K_B/K_A)e^{-i(K_A+K_B)(a+b)} \\ (1 - K_B/K_A)e^{i(K_A+K_B)(a+b)} & (1 + K_B/K_A)e^{i(K_A-K_B)(a+b)} \end{pmatrix}, \quad (40)$$

and

$$M_B^2 = \frac{1}{2} \begin{pmatrix} (1 + K_A/K_B)e^{i(K_A-K_B)(2a+b)} & (1 - K_A/K_B)e^{-i(K_A+K_B)(2a+b)} \\ (1 - K_A/K_B)e^{i(K_A+K_B)(2a+b)} & (1 + K_A/K_B)e^{i(K_B-K_A)(2a+b)} \end{pmatrix}, \quad (41)$$

Finally, we can give the transfer matrices M_A^N and M_B^N of media A and B in the N -th period, they are

$$M_A^N = \frac{1}{2} \begin{pmatrix} (1 + K_B/K_A)e^{i(K_B-K_A)(N-1)(a+b)} & (1 - K_B/K_A)e^{-i(K_A+K_B)(N-1)(a+b)} \\ (1 - K_B/K_A)e^{i(K_A+K_B)(N-1)(a+b)} & (1 + K_B/K_A)e^{i(K_A-K_B)(N-1)(a+b)} \end{pmatrix}, \quad (42)$$

and

$$M_B^N = \frac{1}{2} \begin{pmatrix} (1 + K_A/K_B)e^{i(K_A-K_B)(N(a+b)-b)} & (1 - K_A/K_B)e^{-i(K_A+K_B)(N(a+b)-b)} \\ (1 - K_A/K_B)e^{i(K_A+K_B)(N(a+b)-b)} & (1 + K_A/K_B)e^{i(K_B-K_A)(N(a+b)-b)} \end{pmatrix}. \quad (43)$$

With the transform matrices, we can give their relations:

(a) The representation of the first period transform matrices are

$$\begin{pmatrix} A_y^1 \\ A_{y1}^1 \end{pmatrix} = M_A^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix}, \quad (44)$$

$$\begin{pmatrix} B_y^1 \\ B_{y1}^1 \end{pmatrix} = M_B^1 \begin{pmatrix} A_y^1 \\ A_{y1}^1 \end{pmatrix} = M_B^1 M_A^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix} = M^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix}. \quad (45)$$

(b) The representation of the second period transform matrices are

$$\begin{pmatrix} A_y^2 \\ A_{y2}^2 \end{pmatrix} = M_A^2 \begin{pmatrix} B_y^1 \\ B_{y1}^1 \end{pmatrix} = M_A^2 M_B^1 M_A^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix} = M_A^2 M^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix}, \quad (46)$$

$$\begin{pmatrix} B_y^2 \\ B_{y2}^2 \end{pmatrix} = M_B^2 \begin{pmatrix} A_y^2 \\ A_{y2}^2 \end{pmatrix} = M_B^2 M_A^2 M_B^1 M_A^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix} = M^2 M^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix}. \quad (47)$$

(c) Similarly, the representation of the N -th period transform matrices are

$$\begin{pmatrix} A_y^N \\ A_{yN}^N \end{pmatrix} = M_A^N M_B^{N-1} M_A^{N-1} \dots M_A^2 M_B^1 M_A^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix} = M_A^N M^{N-1} \dots M^2 M^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix}, \quad (48)$$

$$\begin{pmatrix} B_y^N \\ B_y'^N \end{pmatrix} = M_B^N M_A^N M_B^{N-1} M_A^{N-1} \dots M_A^2 M_B^1 M_A^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix} = M^N M^{N-1} \dots M^2 M^1 \begin{pmatrix} F_y \\ F_y' \end{pmatrix} = M \begin{pmatrix} F_y \\ F_y' \end{pmatrix} \quad (49)$$

where

$$M = M^N M^{N-1} \dots M^2 M^1 = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}, \quad (50)$$

is the total transform matrix of N period, and $M^1 = M_B^1 M_A^1$ is the first period transform matrix, $M^2 = M_B^2 M_A^2$ is the second period transform matrix, and $M^N = M_B^N M_A^N$ is the N -th period transform matrix.

The wave function of N -th period in medium B is

$$\begin{aligned} \vec{\psi}_B^N(x, y, z) &= (B_x^N e^{i(K_{Bx}x + K_{Bz}z)} + B_x'^N e^{i(-K_{Bx}x + K_{Bz}z)}) \vec{i} \\ &+ (B_y^N e^{i(K_{Bx}x + K_{Bz}z)} + B_y'^N e^{i(-K_{Bx}x + K_{Bz}z)}) \vec{j} \\ &+ (B_z^N e^{i(K_{Bx}x + K_{Bz}z)} + B_z'^N e^{i(-K_{Bx}x + K_{Bz}z)}) \vec{k}, \end{aligned} \quad (51)$$

In FIG. 1, the transmission wave function is

$$\vec{\psi}_D(x, y, z) = D_x e^{i(K_x x + K_z z)} \vec{i} + D_y e^{i(K_x x + K_z z)} \vec{j} + D_z e^{i(K_x x + K_z z)} \vec{k}. \quad (52)$$

(4) At $x = N(a+b)$, by the \vec{j} component continuation of wave functions $\vec{\psi}_B^N(x, y, z)$ and $\vec{\psi}_D(x, y, z)$, we have

$$B_y^N e^{i(K_{Bx}N(a+b) + K_{Bz}z)} + B_y'^N e^{i(-K_{Bx}N(a+b) + K_{Bz}z)} = D_y e^{i(K_x N(a+b) + K_z z)}, \quad (53)$$

since the Eq. (53) is an equation for an arbitrary variable z , we have

$$B_y^N e^{iK_{Bx}N(a+b)} + B_y'^N e^{-iK_{Bx}N(a+b)} = D_y e^{iK_x N(a+b)}, \quad (54)$$

with Eqs. (49) and (50), the Eq. (54) can be written as

$$(m_1 F_y + m_2 F_y') e^{iK_{Bx}N(a+b)} + (m_3 F_y + m_4 F_y') e^{-iK_{Bx}N(a+b)} = D_y e^{iK_x N(a+b)}, \quad (55)$$

(5) At $x = N(a+b)$, by the \vec{j} component derivative continuation of wave functions $\vec{\psi}_B^N(x, y, z)$ and $\vec{\psi}_D(x, y, z)$, we have

$$K_{Bx} B_y^N e^{iK_{Bx}N(a+b)} - K_{Bx} B_y'^N e^{-iK_{Bx}N(a+b)} = K_x D_y e^{iK_x N(a+b)}, \quad (56)$$

with Eqs. (49) and (50), the Eq. (56) can be written as

$$\frac{K_{Bx}}{K_x} (m_1 F_y + m_2 F_y') e^{iK_{Bx}N(a+b)} - \frac{K_{Bx}}{K_x} (m_3 F_y + m_4 F_y') e^{-iK_{Bx}N(a+b)} = D_y e^{iK_x N(a+b)}. \quad (57)$$

By Eqs. (55) and (57), we can obtain

$$\frac{F_y'}{F_y} = \frac{m_1(K_x - K_{Bx})e^{iK_{Bx}N(a+b)} + m_3(K_x + K_{Bx})e^{-iK_{Bx}N(a+b)}}{m_2(K_{Bx} - K_x)e^{iK_{Bx}N(a+b)} - m_4(K_x + K_{Bx})e^{-iK_{Bx}N(a+b)}}, \quad (58)$$

$$t = \frac{D_y}{F_y} = (m_1 + m_2 \frac{F_y'}{F_y}) e^{i(K_{Bx} - K_x)N(a+b)} + (m_3 + m_4 \frac{F_y'}{F_y}) e^{-i(K_{Bx} + K_x)N(a+b)}, \quad (59)$$

and the quantum transmissivity T is

$$T = |t|^2. \quad (60)$$

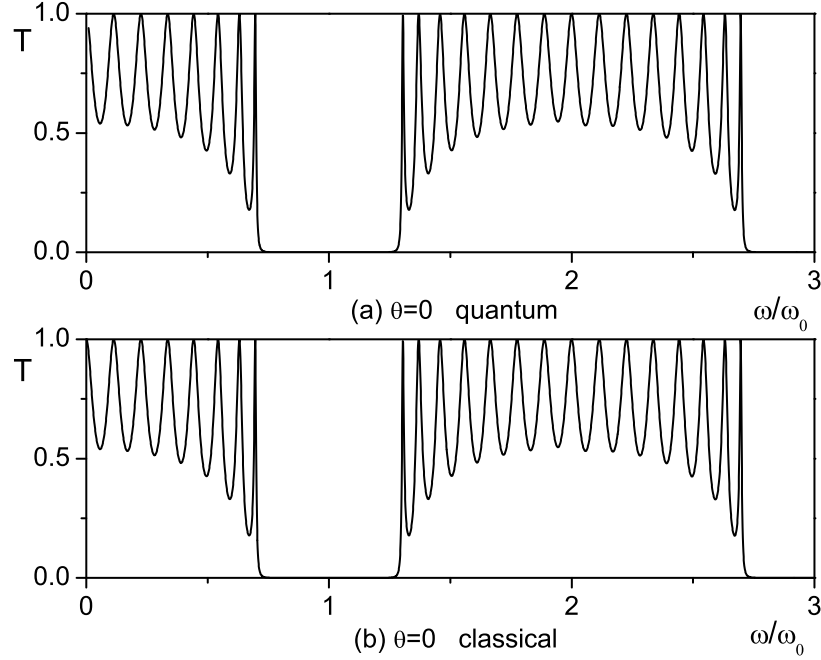


FIG. 2: The quantum and classical transmissivity of incident angle $\theta = 0$. (a) quantum transmissivity (b) classical transmissivity.

5. Numerical result

In this section, we report our numerical results of quantum transmissivity. The main parameters are: For the medium A , its refractive indexes is $n_a = 1.45$, and thickness is $a = 267nm$. For the medium B , its refractive indexes is $n_b = 3.59$, and thickness is $b = 108nm$. The central frequency is $\omega_0 = 1.216 \times 10^{15}Hz$, and the period number is $N = 16$. With Eqs. (58)-(60), we can calculate the quantum transmissivity. In FIG. 2, we calculate the quantum and classical transmissivity when the incident angle $\theta = 0$, FIG. 2 (a) and (b) are quantum and classical transmissivity, respectively. Comparing FIG. 2 (a) and (b), we find the quantum transmissivity and classical transmissivity are the completely same, i.e., when the incident angle $\theta = 0$, there is not quantum effect in one-dimensional photonic crystal. In FIG. 3, we calculate the quantum and classical transmissivity when the incident angle θ are $\frac{\pi}{12}$, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, respectively. From FIG. 3 (a) to (d), they are classical transmissivity. From FIG. 3 (e) to (h), they are quantum transmissivity. We can obtain the following results: (1) With the incident angle increase, the forbidden bands width are unchanged and positions red shift for the classical transmissivity. (2) With the incident angle increase, the forbidden bands become widened and positions blue shift for the quantum transmissivity. (3) For the same incident angle θ , the quantum forbidden bands are wider than the classical forbidden bands, and the quantum forbidden bands positions blue shift. (4) When the incident angle increase, the quantum effect become more remarkable. In FIG. 4, at the incident angle $\theta = \frac{\pi}{6}$, we calculate the quantum and classical transmissivity when the thickness b of medium B are $108nm$ and $158nm$, respectively. FIG. 4 (a) and (b) are the classical transmissivity, and FIG. 4 (c) and (d) are the quantum transmissivity. We can obtain the following results: (1) With the thickness b increase, the forbidden bands numbers increase and positions red shift for the classical and quantum transmissivity. (2) For the same thickness b , the quantum forbidden bands are wider than the classical forbidden bands, and the quantum forbidden bands positions blue shift relative to the classical forbidden bands. In FIG. 5, at the incident angle $\theta = \frac{\pi}{6}$, we calculate the quantum and classical

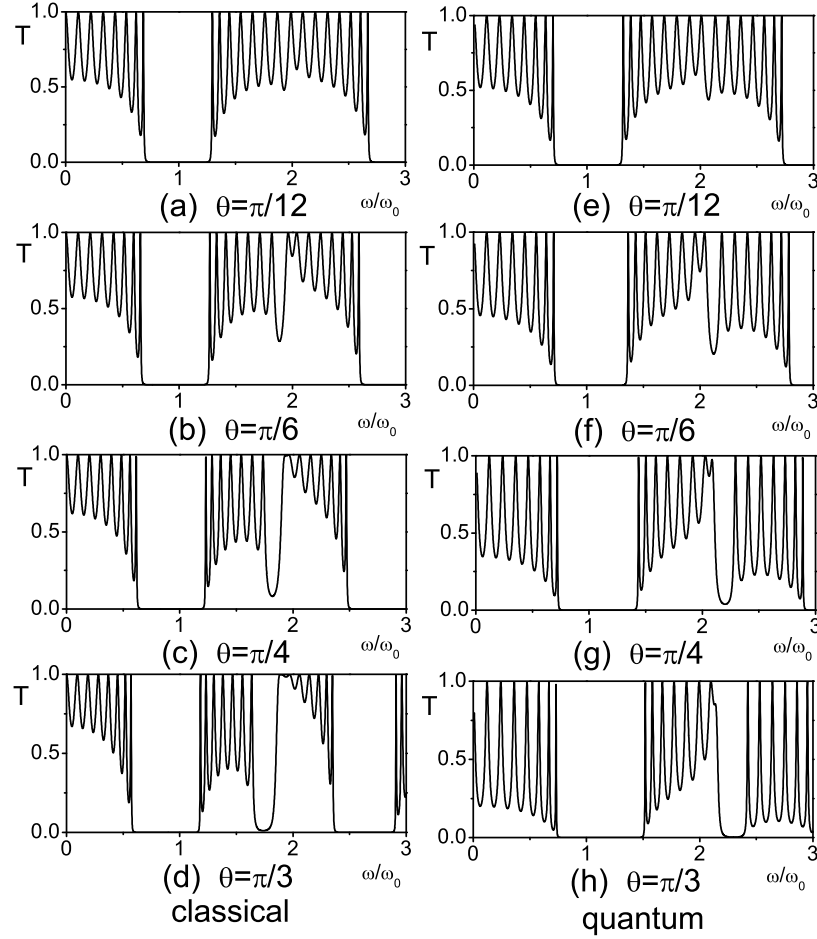


FIG. 3: The quantum and classical transmissivity for different incident angle θ . (a)-(d) classical transmissivity, (e)-(h) quantum transmissivity.

transmissivity when the refractive indexes n_b of medium B are 3.59 and 4.09, respectively. FIG. 5 (a) and (b) are the classical transmissivity, and FIG. 5 (c) and (d) are the quantum transmissivity. We can obtain the following results: (1) With the refractive indexes n_b increase, the forbidden positions red shift for the classical and quantum transmissivity. (2) For the same refractive indexes n_b , the quantum forbidden bands are wider than the classical forbidden bands, and the quantum forbidden bands positions blue shift relative to the classical forbidden bands. In FIG. 6, we calculate the quantum and classical transmissivity with defect layer, and the incident angle $\theta = 0$, FIG. 6 (a) and (b) are quantum and classical transmissivity, respectively. Comparing FIG. 6 (a) and (b), we find the classical and quantum transmissivity are identical, i.e., when the incident angle $\theta = 0$, there is not quantum effect in one-dimensional photonic crystal with defect layer. In FIG. 7, we calculate the classical and quantum transmissivity with defect layer, and the incident angle are $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, respectively, and the structure is $(AB)^8 D (AB)^8$. From FIG. 7 (a) to (c), they are classical transmissivity. From FIG. 7 (d) to (f), they are quantum transmissivity. We can obtain the following results: (1) With the incident angle increase, the forbidden bands width and defect model intensity are unchanged, but the forbidden bands and defect model positions red shift for the classical transmissivity. (2) With the

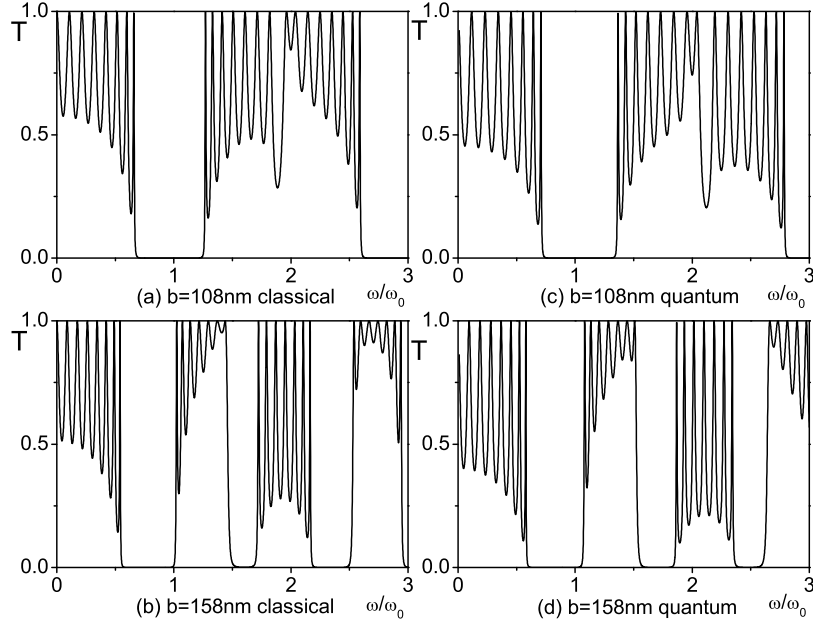


FIG. 4: The quantum and classical transmissivity of incident angle $\theta = \frac{\pi}{6}$ for different thickness b . (a)-(b) classical transmissivity, (c)-(d) quantum transmissivity.

incident angle increase, the quantum forbidden bands become widened, the defect model intensity weaken, and the forbidden bands and defect model positions blue shift relative to the classical transmissivity. (3) For the same incident angle, the quantum forbidden bands are wider and defect model intensity weaker than the classical forbidden bands, the defect model and the quantum forbidden bands positions blue shift relative to the classical. (4) When the incident angle increase, the quantum effect become more remarkable.

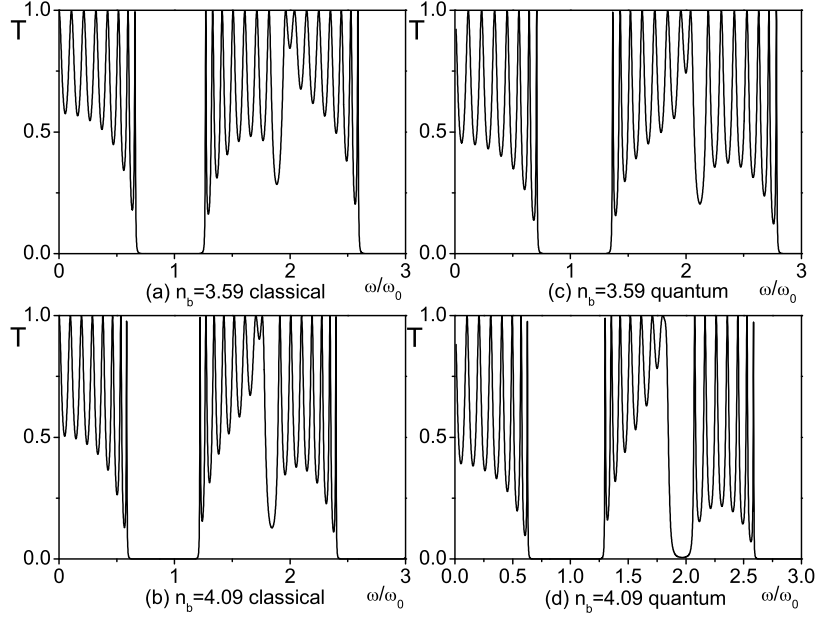


FIG. 5: The quantum and classical transmissivity of incident angle $\theta = \frac{\pi}{6}$ for different refractive indexes n_b . (a)-(b) classical transmissivity, (c)-(d) quantum transmissivity.

6. Conclusion

In summary, we have studied the quantum transmission characteristics of one-dimensional photonic crystal by the quantum theory approach, and compared the calculation results of classical with quantum theory. We have found some quantum effects in one-dimensional photonic crystal. When the incident angle $\theta = 0$, i.e., vertical incidence, the classical and quantum transmission characteristics are the completely same, i.e., there is not quantum effect in one-dimensional photonic crystal. When the incident angle $\theta \neq 0$, we find there are obvious quantum effect with the incident angle increase. At the incident angle $\theta \neq 0$, there are also obvious quantum effect with the change of thickness and refractive indexes of medium B or A . Otherwise, we have considered the effect of defect layer on the quantum transmission characteristics. When the incident angle $\theta = 0$, there is also not quantum effect, and when the incident angle $\theta \neq 0$, with the incident angle increase, there are obvious quantum effect for the one-dimensional photonic crystal with defect layer. The new quantum effect of photonic crystal shall help us to design optical devices.

7. Acknowledgment

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¹ E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).

² S. John, Phys. Rev. Lett. **58** 2486 (1987).

³ A. Lavrinenko, P.I. Borel, L.H. Frandsen, M. Thorhauge, A. Harph, M. Kristensen, T. Niemi, Opt. Express **12** 234 (2004).

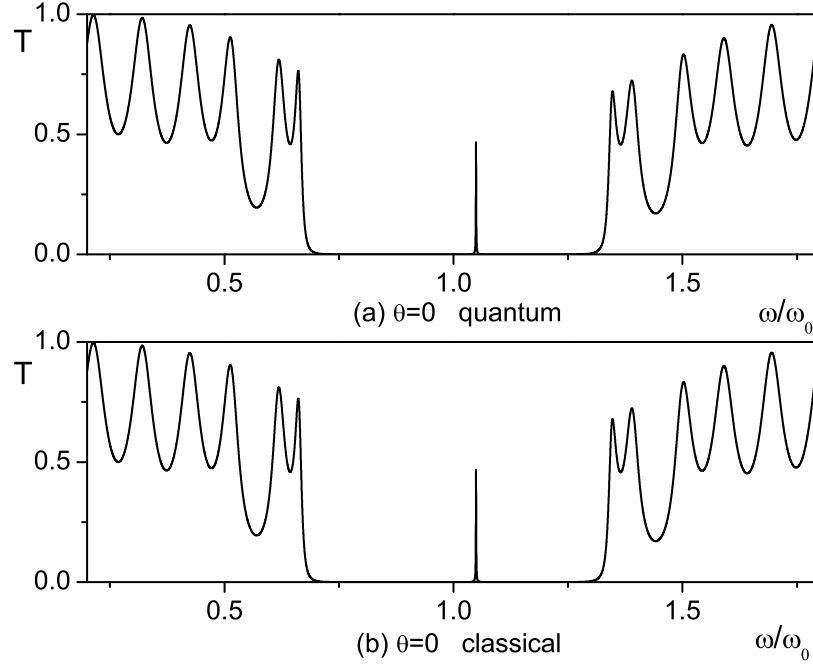


FIG. 6: The quantum and classical transmissivity of incident angle $\theta = 0$ with defect layer, the structure is $(AB)^8D(AB)^8$. (a) quantum transmissivity (b) classical transmissivity.

- ⁴ J. Pu, Y. Yomogida, K. K. Liu, L. J. Li, Y. Iwasa, and T. Takenobu, *Nano Lett.*, **12** 4013, (2012).
- ⁵ S. Fan, P.R. Villeneuve, J.D. Joannopoulos, H.A. Haus, *Phys. Rev. Lett.* **80** 960 (1998).
- ⁶ A. Ferreira, N. M. R. Peres, R. M. Ribeiro and T. Stauber, *Phys. Rev. B*, **85** 115438, (2012).
- ⁷ S. Kim, I. Park, H. Lim, *Proc. Design of photonic crystal splitters/combiners SPIE* **5597** 129 (2004).
- ⁸ N. M. R. Peres and Yu. V. Bludov, *EPL*, **101** 58002, (2013).
- ⁹ R. Martinez-Sala, J. Sancho, J. V. Sanchez, V. Gomez, J. Llinares and F. Meseguer, *nature* **378**, 241 (1995).
- ¹⁰ T. Cai, R. Bose, G. S. Solomon, and E. Waks, *Appl. Phys. Lett.* **102** 141118 (2013).
- ¹¹ J. J. Joannopoulos, R. D. Meade, J. N. Winn, *Photonic crystals: molding the flow of light* (Princeton University Press, New Jersey, 1995).
- ¹² M. Minkov and V. Savona, *Phys. Rev. B* **88** 081303R (2013).
- ¹³ K. K. Yee, *IEEE Trans. Antennas Propag.* **14**, 302 (1966).
- ¹⁴ K. S. Choi, H. Deng, J. Laurat, and H. J. Kimble, *Nature* **452**, 67 (2006).
- ¹⁵ J. B. Pendry, *Phys. Rev. Lett.* **69**, 2772 (1992).
- ¹⁶ M. Jachura, M. Karpinski, C. Radzewicz, and K. Banaszek, *Opt. Express* **22** 8624 (2014).
- ¹⁷ D. A. Miller, *Nature Photonics* **4** 3 (2010).
- ¹⁸ Kamal, A., Clarke, J. Devoret, M. H, *Nature Physics*, **7** 311 (2011).
- ¹⁹ Fan, L., *Science*, **335** 447 (2012).
- ²⁰ Yu Z., Fan S., *Nature Photon.* **3** 91 (2009).
- ²¹ Xiang-Yao Wu, Xiao-Jing Liu, and Yi-Heng Wu, et. al., *Int J Theor Phys*, **49**, 194 (2010).
- ²² B. J. Smith and M. G. Raymer, *Photon wave functions [J]New J. Phys.* **9**, 414 (2007).
- ²³ Xiang-Yao Wu, Ji Ma, Xiao-Jing Liu, Jing-Hai Yang, Hong Li, Si-Qi Zhang, Hai-Xin Gao, Xin-Guo Yin, San Chen, *Physica E* **59**, 174 (2014).
- ²⁴ Xiang-Yao Wu, Ji Ma, Xiao-Jing Liu, Jing-Hai Yang, Hong Li, Si-Qi Zhang, Hai-Xin Gao, Heng-Mei Li, Hong-Chun Yuan, *Optics Communications* **321**, 211 (2014).

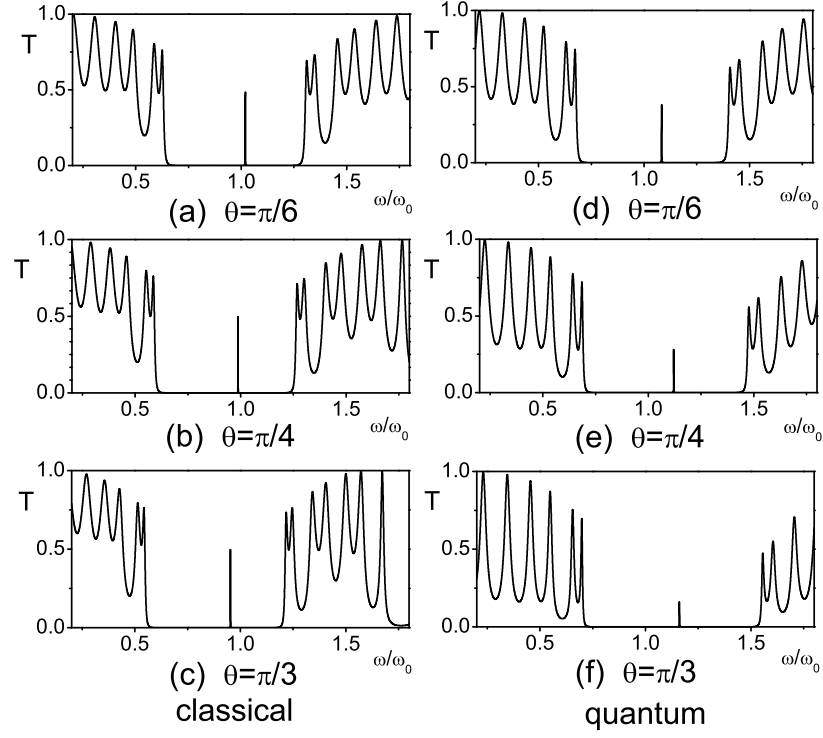


FIG. 7: The quantum and classical transmissivity for different incident angle θ with defect layer, the structure is $(AB)^8D(AB)^8$. (a)-(c) classical transmissivity, (d)-(f) quantum transmissivity.